Int J Theor Phys (2010) 49: 657–659
DOI 10.1007/s10773-009-0225-5

Comment on "Local Copying of $d \times d$-dimensional Partially Entangled Pure States"

Ramij Rahaman

Received: 31 July 2009 / Accepted: 9 December 2009 / Published online: 15 December 2009

Abstract Recently, Li et al. (Int. J. Theor. Phys. 48:2777, 2009) derived a necessary and sufficient condition for LOCC cloning of a set of bipartite orthogonal partially but equally entangled state. We demonstrates that, the result is based on a wrong observation regarding a set of non-maximally entangled states with equal entanglement. We also provide a simple example in favor of our comment.

Keywords Local copying · Partially entanglement · Unitary operator

1 Introduction

In a recent publication by Li and Shen [1] derive the necessary and sufficient condition for local copying[1] of a set of bipartite orthogonal partially but equally entangled (BOPEE) states. To established the result they claim to have derived a relation to hold for a set of BOPEE states. Let $\{|\Psi_j\rangle\}_{j=1}^{n}$ be a set of BOPEE states. According to their claim the states can be expressed as $|\Psi_j\rangle = (U_j^1 \otimes I^2)|\Psi_1\rangle$ (U_j's are the unitary operator acting on first system and I is the identity operator acting on the second system). But this is not true in general. Though the results of [1] have no contradiction with the necessary condition given in [2, 3] for Local copying of a set of BOPEE states in various cases. In particular, for maximally entangled states, the above relation is true [4–6], whereas this may not hold even for a pair of BOPEE states.

To show this we consider the following pair of states $|\Psi_1\rangle = a|00\rangle + b|11\rangle$ and $|\Psi_2\rangle = b^*|00\rangle - a^*|11\rangle$, where '$*$' indicate the complex conjugate and $|a|^2 + |b|^2 = 1;\ |a| \neq |b|;\ 0 < |a|, |b| < 1$. Here $\{|\Psi_j\rangle\}_{j=1}^{2}$ is a set of two BOPEE states. Now we show that $|\Psi_2\rangle$ does not satisfy the relation $|\Psi_2\rangle = (U_2^1 \otimes I^2)|\Psi_1\rangle$, for any unitary U_2^1 acting on first Hilbert space $\mathcal{H}^1$.

[1] Cloning under local operation and classical communication (LOCC).

R. Rahaman (✉)
Selmer Center, Department of Informatics, University of Bergen, Bergen, Norway
e-mail: Ramij.Rahaman@ii.uib.no

If possible, we assume that, there exists a 2×2 unitary operator such that $|\Psi_2\rangle = (U \otimes I)|\Psi_1\rangle$ hold.

The general form of a 2×2 unitary matrix is $U = \begin{pmatrix} \alpha & \lambda\beta \\ -\beta^* & \lambda\alpha^* \end{pmatrix}$, where α, β, λ are complex and $|\alpha|^2 + |\beta|^2 = 1 = |\lambda|$.

If $|\Psi_2\rangle = (U \otimes I)|\Psi_1\rangle$ holds, then from simple algebra we have the following equations.

$$a\alpha = b^* \tag{1}$$

$$b\lambda\beta = 0 \tag{2}$$

$$-a\beta^* = 0 \tag{3}$$

$$b\lambda\alpha^* = -a^* \tag{4}$$

From (2) and (3) we have,

$$\beta = 0 \quad (\text{since, } a \neq 0 \neq b \text{ and } |\lambda| = 1) \tag{5}$$

Therefore, $|\alpha| = 1$.

Now (1) and (4) have solution only if $|a| = |b|$, as, $|\alpha| = 1 = |\lambda|$. $|a| = |b|$ imply that both $|\Psi_1\rangle$ & $|\Psi_2\rangle$ are maximally entangled states.

Therefore, for non-maximal state, (1–4) are inconsistent, which imply that $|\Psi_2\rangle$ can't be expressed as $|\Psi_2\rangle = (U \otimes I)|\Psi_1\rangle$, for any 2×2 unitary operator U.

Let us now point out the wrong step in their [1] derivation, which led them to this wrong result.

Let $\{|e_i\rangle\}_{i=1}^d$ be an orthogonal basis for single particle Hilbert space $\mathcal{H}$ of dimension d. $|\Phi_1\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^d |e_i\rangle|e_i\rangle$ is a maximally entangled state in $\mathcal{H}^{\otimes 2}$ and $|\Psi_j\rangle = \sum_{i=1}^d \alpha_i^j |e_i\rangle|e_i\rangle$ are the non-maximally entangled states in $\mathcal{H}^{\otimes 2}$, for $j = 1, 2, \ldots, n$, with $\sum_{i=1}^d |\alpha_i^j|^2 = 1$.

Now it is true that for every pure bipartite non-maximally entangled state $|\Psi_1\rangle$, there exist, a POVM's[2] $\mathcal{M}^1$ acting on the first Hilbert space $\mathcal{H}^1$, such that

$$|\Psi_1\rangle = (\mathcal{M}^1 \otimes I^2)|\Phi_1\rangle \tag{6}$$

Let $\{|\Psi_j\rangle\}_{j=1}^n$ be a set of BOPEE states. Then the following relation holds

$$|\Psi_j\rangle = (V_j^1 \otimes W_j^2)|\Psi_1\rangle \tag{7}$$

for all j $(= 1, 2, \ldots, n)$, V_j^1 and W_j^2 being unitary on $\mathcal{H}^1$ and $\mathcal{H}^2$ respectively.

In particular $V_1^1 = I^1$ and $W_1^2 = I^2$. From (6) and (7), we have

$$|\Psi_j\rangle = (V_j^1 \otimes W_j^2)(\mathcal{M}^1 \otimes I^2)|\Phi_1\rangle \tag{8}$$

Li et al. has rewritten (8) as

$$|\Psi_j\rangle = (\mathcal{M}^1 \otimes I^2)(V_j^1 \otimes W_j^2)|\Phi_1\rangle \tag{9}$$

from which the desired relation

$$|\Psi_j\rangle = (I^1 \otimes U_j^2)|\Psi_1\rangle \tag{10}$$

[2] Positive Operator-Valued Measure.

(or, equivalently $|\Psi_j\rangle = (U_j^1 \otimes I^2)|\Psi_1\rangle$ for $j = 1, 2, \ldots, n$) follows. But the problem is that (9) may not follow from (8) as $(\mathcal{M}^1 \otimes I^2)$ and $(V_j^1 \otimes W_j^2)$ may not commute in general.

2 Conclusion

In conclusion, we have pointed out an error in the derivation of a result needed to prove a theorem on local cloning of orthogonal entangled states given in [1]. The interesting problem of finding the necessary and sufficient condition for local copying of arbitrary set of bipartite orthogonal partially but equally entangled states still remains open.

Acknowledgement The research has been supported by Norwegian Research Council. The author is grateful to G. Kar and S. Ghosh for fruitful discussions.

References

1. Li, D., Shen, Z.: Local copying of $d \times d$-dimensional partially entangled pure states. Int. J. Theor. Phys. **48**, 2777 (2009)
2. Choudhary, S.K., Kunkri, S., Rahaman, R., Roy, A.: Local cloning of entangled qubits. Phys. Rev. A **76**, 052305 (2007)
3. Kay, A., Ericsson, M.: Local cloning of arbitarily entangled multipartite states. Phys. Rev. A **73**, 012343 (2006)
4. Ghosh, S., Kar, G., Roy, A.: Local cloning of Bell states and distillable entanglement. Phys. Rev. A **69**, 052312 (2004)
5. Anselmi, F., Chefles, A., Plenio, M.: Local copying of orthogonal quantum states. New J. Phys. **6**, 164 (2004)
6. Owari, M., Hayashi, M.: Local copying and local discrimination as a study for nonlocality of a set of states. Phys. Rev. A **74**, 032108 (2006)